while

$$\left\lfloor \sqrt{4n + 4\sqrt[3]{n} + 2} \right\rfloor \;=\; \lfloor 14.008\cdots \rfloor = 14\,.$$

It also fails for $n = 95$ and 616.

Hawkins and Stone also gave the following comment: It is straightforward to show that the function $\sqrt{n} + \sqrt{n + 2\sqrt[3]{n} + 1}$ *is less than* $\sqrt{4n + 4\sqrt[3]{n} + 2}$*, and the gap between them narrows as* n *increases. But sometimes the values "straddle" an integer, as shown above, so the desired equality fails. We suspect that happens infinitely often (the next instance is* 1249*).*

SALEM MALIKIĆ, student, Sarajevo College, Sarajevo, Bosnia and Herzegovina, also gave the counterexample $n = 45$*. There were three incorrect "proofs" for the assertion.*

3241. [2007 : 237, 239] *Proposed by Virgil Nicula, Bucharest, Romania.*

Let a, b, c be any real numbers such that $a^2 + b^2 + c^2 = 9$. Prove that

$$3 \cdot \min\{a, b, c\} \;\le\; 1 + abc\,.$$

Solution by Arkady Alt, San Jose, CA, USA, modified by the editor.

Without loss of generality, we may assume $a \le b \le c$. We want to prove the inequality $abc + 1 \ge 3a$.

If $a \le 0$, then using the inequality $bc \le \frac{1}{2}(b^2 + c^2)$, we obtain

$$\begin{aligned} abc + 1 - 3a \;&\ge\; a|bc| - 3a + 1 \;\ge\; \tfrac{1}{2}a(b^2 + c^2) - 3a + 1 \\ &=\; \tfrac{1}{2}a(9 - a^2) - 3a + 1 \;=\; \tfrac{1}{2}(a+1)^2(2 - a) \;\ge\; 0\,, \end{aligned}$$

with equality if and only if $a = -1$ and $b = c = 2$.

Now, let $a > 0$. Since $a \le b \le c$, we get $9 = a^2 + b^2 + c^2 \ge 3a^2$; whence, $a \le \sqrt{3}$. We have the obvious inequality $(c^2 - a^2)(b^2 - a^2) \ge 0$, which yields

$$bc \;\ge\; a\sqrt{c^2 + b^2 - a^2} \;=\; a\sqrt{9 - 2a^2}\,.$$

Hence, it suffices to prove the (stronger) inequality

$$a\bigl(a\sqrt{9 - 2a^2}\,\bigr) + 1 \;\ge\; 3a\,.$$

If $0 < a \le 1$, then $\sqrt{9 - 2a^2} \ge \sqrt{7} > \frac{9}{4}$; thus, $4a^2\sqrt{9 - 2a^2} > 9a^2$. Using the inequality $(t+1)^2 \ge 4t$, we obtain

$$\bigl(a^2\sqrt{9 - 2a^2} + 1\bigr)^2 \;\ge\; 4a^2\sqrt{9 - 2a^2} \;>\; 9a^2\,,$$

which yields

$$a\bigl(a\sqrt{9 - 2a^2}\,\bigr) + 1 \;\ge\; 3a\,.$$

If $1 < a \le \sqrt{3}$, then we can prove an equivalent form of our inequality, namely, the one obtained by squaring both sides of

$$a\bigl(a\sqrt{9 - 2a^2}\,\bigr) \;\ge\; 3a - 1\,;$$

that is,

$$2a^6 - 9a^4 + 9a^2 - 6a + 1 \leq 0 .$$

We have

$$\begin{aligned} 2a^6 - 9a^4 + 9a^2 - 6a + 1 &< 2a^6 - 9a^4 + 9a^2 - 5 \\ &= (2a^2 - 1)(a^2 - 1)(a^2 - 3) - (a^2 + 2) \\ &< 0 , \end{aligned}$$

which completes the proof.

Also solved by MOHAMMED AASSILA, Strasbourg, France; ROY BARBARA, Lebanese University, Fanar, Lebanon; JOHN HAWKINS and DAVID R. STONE, Georgia Southern University, Statesboro, GA, USA; WALTHER JANOUS, Ursulinengymnasium, Innsbruck, Austria; KEE-WAI LAU, Hong Kong, China; SALEM MALIKIĆ, student, Sarajevo College, Sarajevo, Bosnia and Herzegovina; ALEX REMOROV, student, William Lyon Mackenzie Collegiate Institute, Toronto, ON; PETER Y. WOO, Biola University, La Mirada, CA, USA; and the proposer.

3243. [2007 : 237, 239] *Proposed by G. Tsintsifas, Thessaloniki, Greece.*

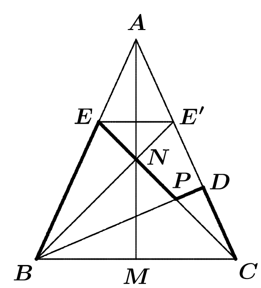
Let ABC be an isosceles triangle with $AB = AC$, and let P be an interior point. Let the lines BP and CP intersect the opposite sides at the points D and E, respectively. Find the locus of P if

$$PD + DC = PE + EB .$$

Solution by Titu Zvonaru, Cománeşti, Romania.

The locus consists of the points inside the triangle on the altitude from A.

Note that since the triangle is isosceles, the foot, M, of the altitude is the mid-point of BC, and the triangle is symmetric about AM. Without loss of generality, label the figure so that $BE \geq CD$ and let E' be the point between A and C for which $CE' = BE$, and let BE' and CE intersect at N. Because $\triangle ABC$ is isosceles, $BCE'E$ is an isosceles trapezoid that is symmetric about AM; whence, N belongs to AM and $NE = NE'$. As a consequence, the following statements are equivalent:



$$\begin{aligned} PD + DC &= PE + EB = PN + NE + E'C \\ &= PN + NE' + E'D + DC , \\ PD &= PN + NE' + E'D . \end{aligned}$$

The last equality says that the quadrilateral $PDE'N$ is degenerate; that is, D coincides with E' and P with N, so that P lies on AM. The locus of P is therefore the open line segment AM, as claimed.